Black Holes: The Laboratory for Quantum Gravity

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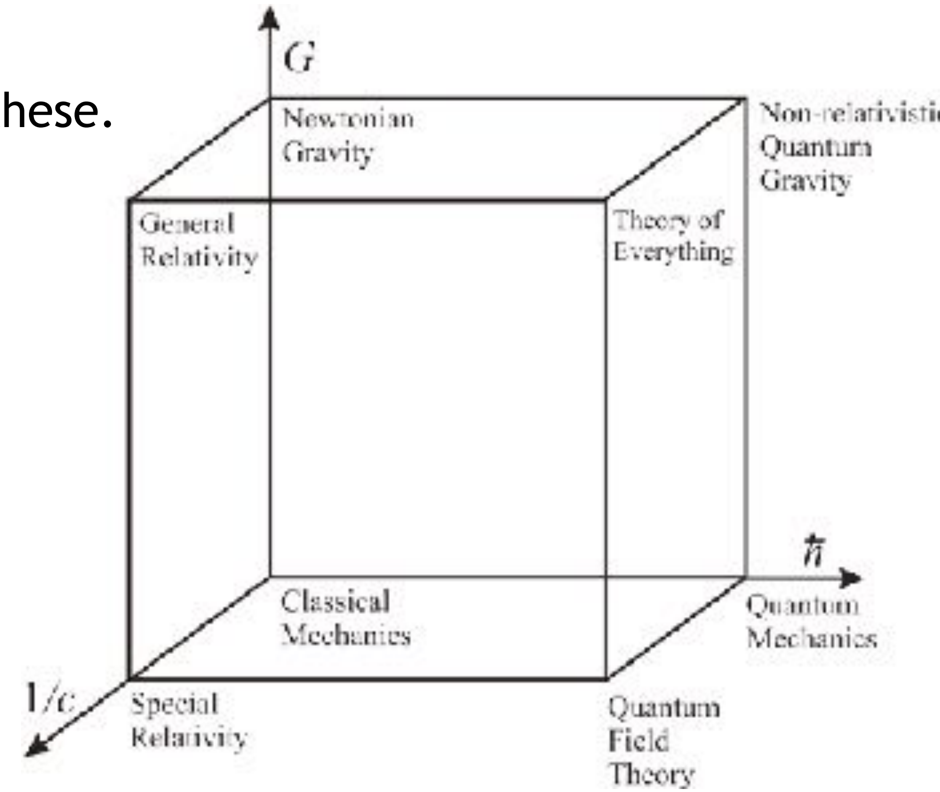
The physics as we know today contains three fundamental constants:
Speed of light c, Planck constant h and Gravitational constant G

The Planck Length is a dimension
of length we can construct with these.

$$l_p = \left(\frac{G\hbar}{c^3} \right)^{1/2} \sim 10^{-33} cm$$



The physics at Planck scale is named as Quantum Gravity and
we have almost no idea of this…..

Why we need Quantum theory of gravity:

We have reasonable quantum description of all basic interactions except gravity, The quest for a Quantum theory of Gravity is quite natural in the overall scheme of unification.

The Quantum Theory of Strings shows that a theory of Quantum gravity may automatically be a theory of all physical interactions.

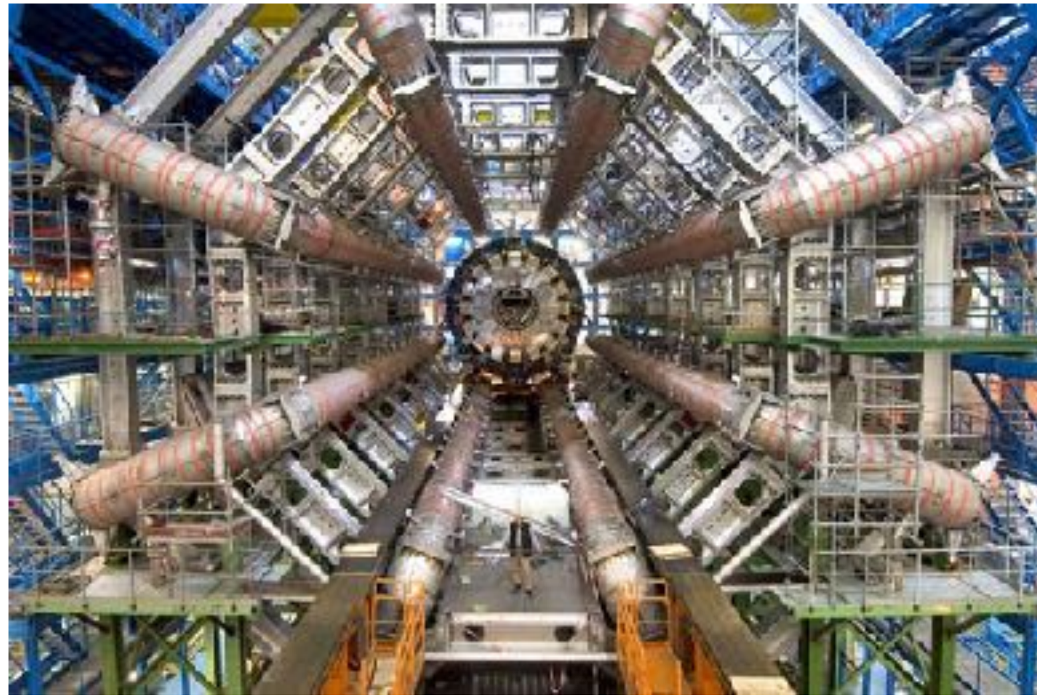
The classical theory of gravity has problematic features like space time singularities at the beginning of the universe and the centre of a black hole

Semi classical gravity also leads to conceptual problems like information paradox.

Quantum gravity is highly desirable

But, the effects of Quantum gravity are tiny, beyond the reach of any near future technology.

Large Hadron Collider:



LHC can see up to a length scale $\sim 10^{-19} m \sim 10^{4} GeV$

Planck Scale $\sim 10^{-35} m \sim 10^{19} GeV$

The unobservability of Quantum Gravity effects creates philosophical objections against the quest…..

How will we compare different models of quantum theory of gravity?

What are the basic criteria for being a successful theory of quantum gravity?

The physics of the black holes comes in rescue:

Quantum Gravity:

$\hbar \to 0$ $\longrightarrow$ Einstein Theory of General Relativity

General relativity describes gravity in terms of space time curvature associated with a metric:

$$ds^2 = g_{ab}(x^m)\, dx^a dx^b$$

In the absence of matter, the metric is flat, Minkowskian.

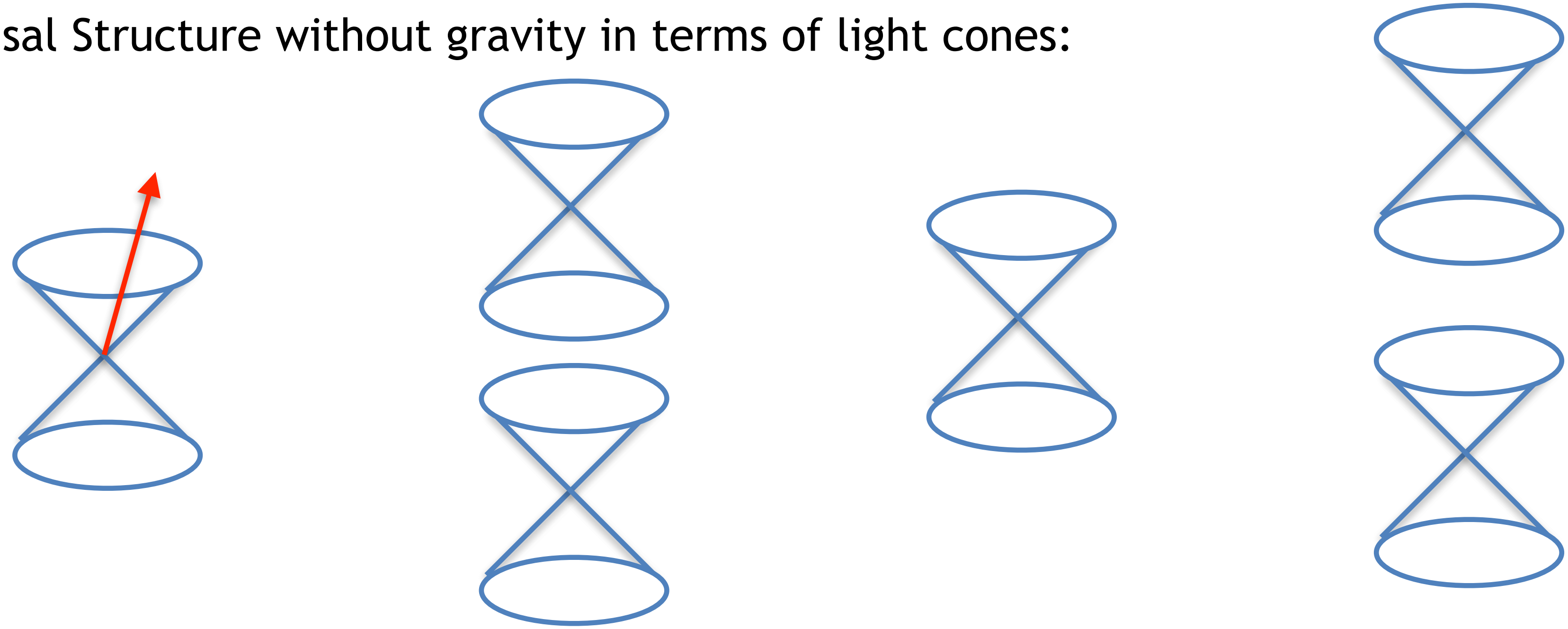
$$ds^2 = -c^2 dt^2 + dx^2 + dy^2 + dz^2$$

In the presence of matter, the metric is determined by solving the Einstein's field equations.

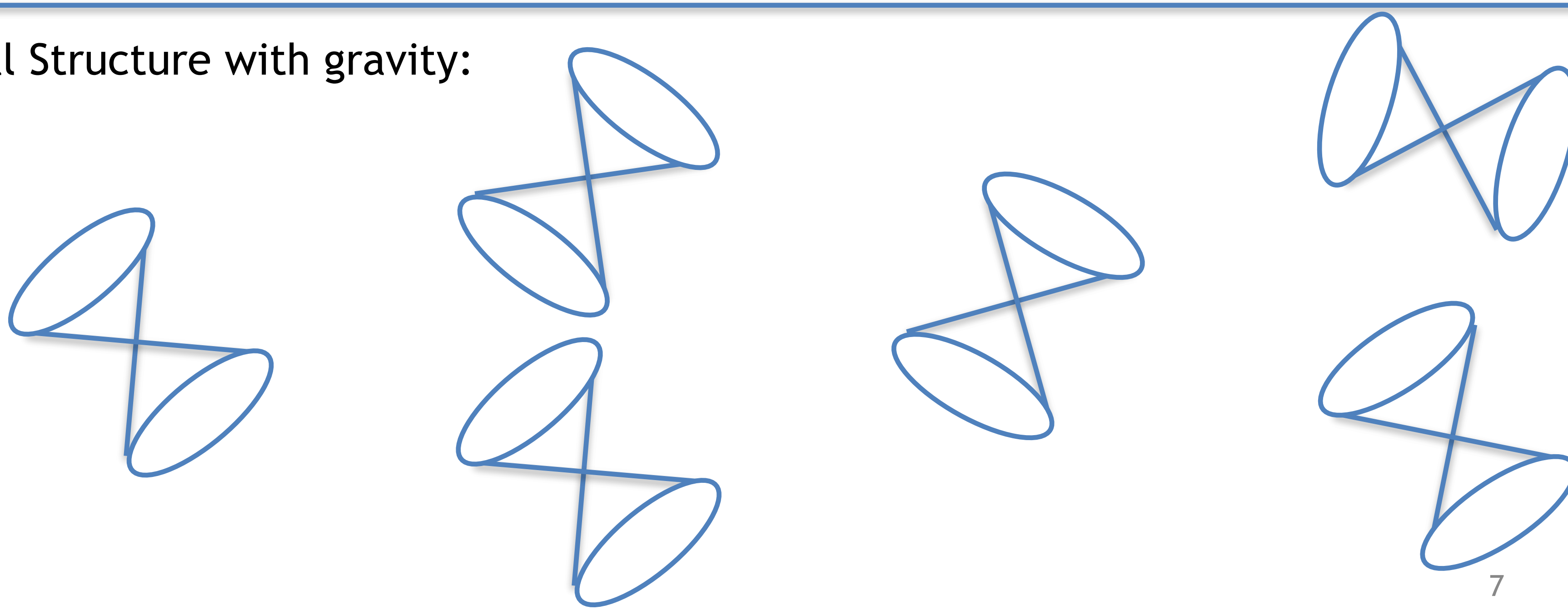
$$G_{ab} = 8\pi \mathrm{G}\, \mathrm{T}_{ab}$$

Geometry = Matter

Causal Structure without gravity in terms of light cones:



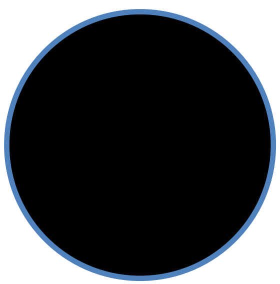
Causal Structure with gravity:



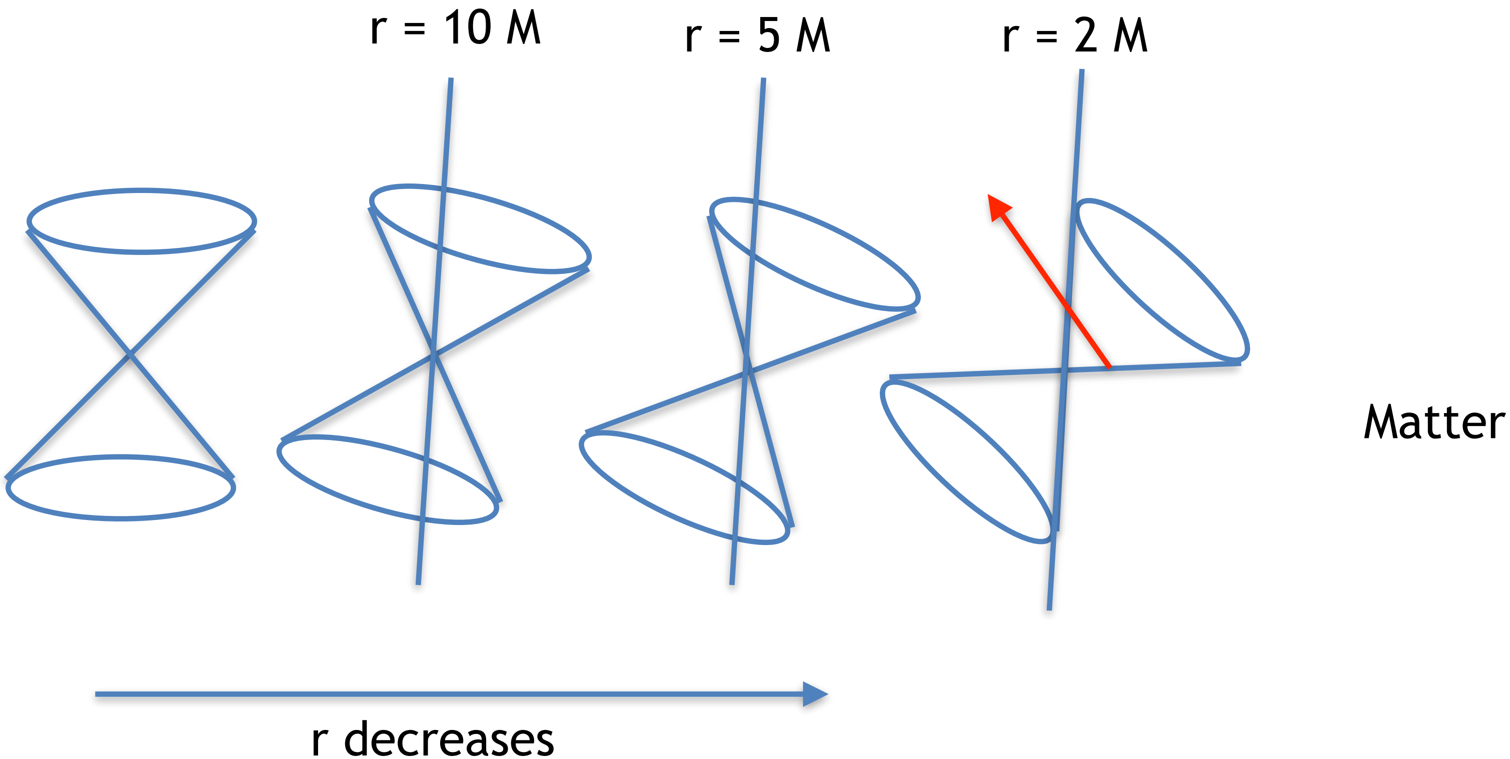
A Black hole is a solution of Einstein's equation with a causal structure, such that beyond a boundary nothing can come back….

Schwarzschild solution:

$$ds^2 = -\left(1 - \frac{2M}{r}\right)dt^2 + \frac{dr^2}{\left(1 - \frac{2M}{r}\right)} + r^2 d\Omega^2$$



Represents the gravitational field outside a specially symmetric matter distribution.



The event horizon at r = 2M is a causal boundary. An outside observer has no access to inside information.

The causal boundary in a black hole space time is called the event horizon. It is a one way membrane hiding information about the interior from the outside world.

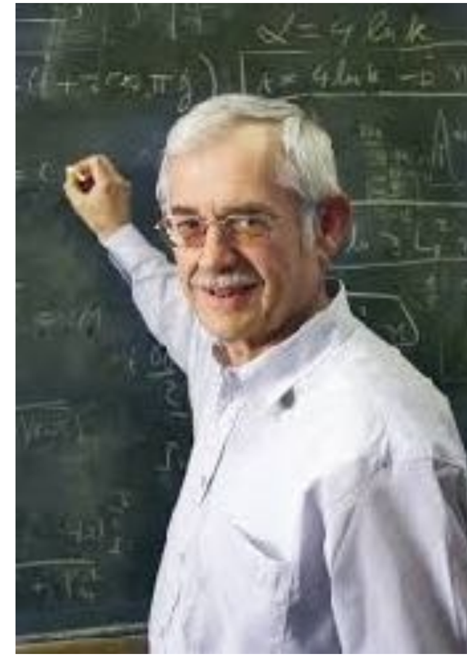
$$ds^2 = -\left(1 - \frac{2GM}{c^2 r}\right) dt^2 + \frac{dr^2}{\left(1 - \frac{2GM}{c^2 r}\right)} + r^2 d\Omega^2$$

Event Horizon is at: $r_h = \frac{2GM}{c^2}$

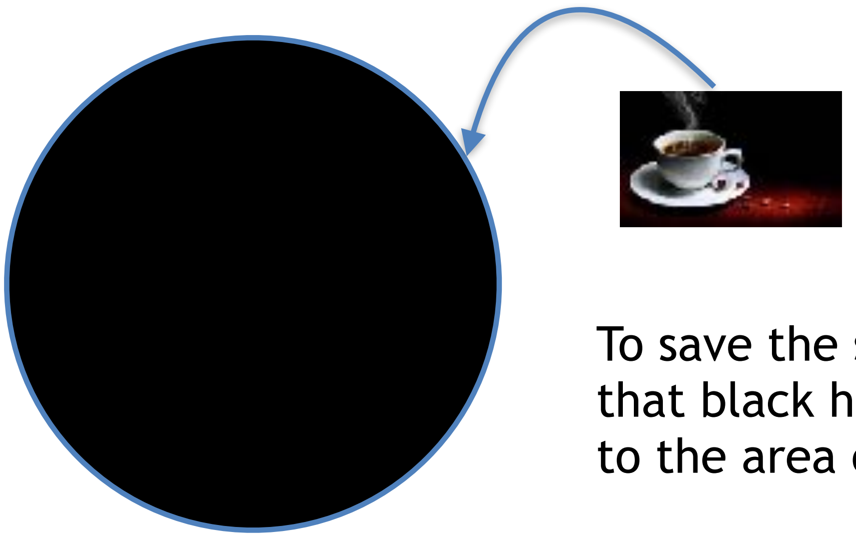
The centre is the singularity where the curvature diverges leading to infinite tidal force....

$$R_{abcd} R^{abcd} = \frac{48 G^2 M^2}{c^4 r^6}$$

Black hole entropy: Bekenstein’s great idea!



Violation of second law outside a black hole.



To save the second law, Bekenstein proposed that black holes must have entropy proportional to the area of the horizon.

After all, black holes block information!

But why black hole entropy is proportional to area:

A naive argument:

Consider a black hole formed from the collapse of a star of mass M.

Assume, one bit of information loss per atom (of mass m). Then the naïve estimate of entropy would be M/m. In principle, there is no bound on `m' from classical mechanics, so use naïve QM, the Compton wavelength of the atom must be less than the radius of the hole

$$S \sim \frac{M}{m} \sim \frac{M r_h}{\hbar}$$

Use the Schwarzschild solution to conclude $r_h \sim M \Rightarrow S \sim r_h^2$

Black hole entropy in GR is proportional to the area!

This is a good news because in classical GR, we have the area theorem:

Area of the event horizon can not decrease as long as:

1. Matter energy is positive.

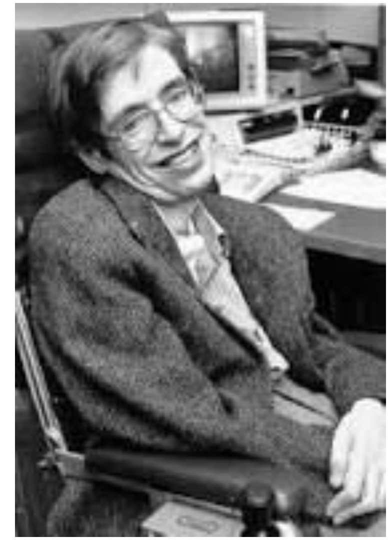
2. Some form of cosmic censorship is valid.

The area theorem ensures that when a entropic objects falls into the black hole, the outside world looses entropy but the black hole area increases to compensate the loss and thereby saves the second law.

The statement is called the generalized second law. (GSL).

But if black holes have entropy, it should have a temperature.

Black Hole Thermodynamics:



In Classical General relativity, area-entropy relationship is still an analogy.

But, the Hawking's semi classical derivation of Black hole temperature completes the thermodynamic interpretation.

Black holes are indeed hot! and GSL is nothing but the statement of ordinary second law of thermodynamics in the presence of a black hole.

$$T_{BH} = \frac{\hbar c^3}{8\pi G K_B M} \qquad S = \frac{c^3 A}{4G\hbar}$$

Black hole Thermodynamics: $T_H \delta S = \delta M$

In fact, the applicability of black hole thermodynamics transcends general relativity and application of extended classical theory of gravities. (See My Papers)

What does it mean that black hole are hot.

Consider a ensemble of two state systems such that initially all of these are in ground state.

E_2

and N copies

E_1

Place these near a collapsing object forming a black hole and wait for long.

Then, some of these systems will be in round state and some in exited state.
A calculation in semi classical gravity shows:

$$\frac{N_2}{N_1} \sim Exp\left(-\frac{E_2 - E_1}{k_B T_{BH}}\right) \qquad T_{BH} = \frac{\hbar c^3}{8\pi G K_B M}$$

The true meaning of entropy is in the statistical interpretation and identification of the micro states.

Understanding the statistical origin of black hole entropy is still an open problem.

In string theory, it is possible to identify objects and configurations, known as D-branes, which corresponds to (not Schwarzschild) black holes in the classical limit, the counting of the degeneracy of states of such configurations exactly produces the Bekenstein entropy including the correct factor.

The derivation of black hole entropy is a basic consistency check for any proposal of Quantum Gravity.

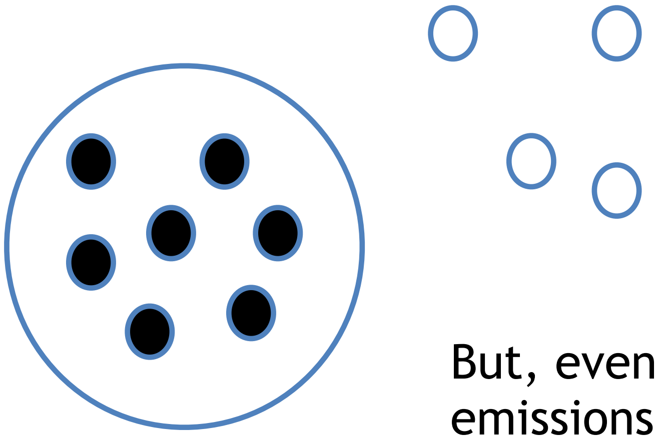
$$\Omega_{BH} \sim Exp(S) \qquad\qquad S = \frac{c^3 A}{4G\hbar}$$

Hawking Radiation is purely thermal and that leads to a Big problem.

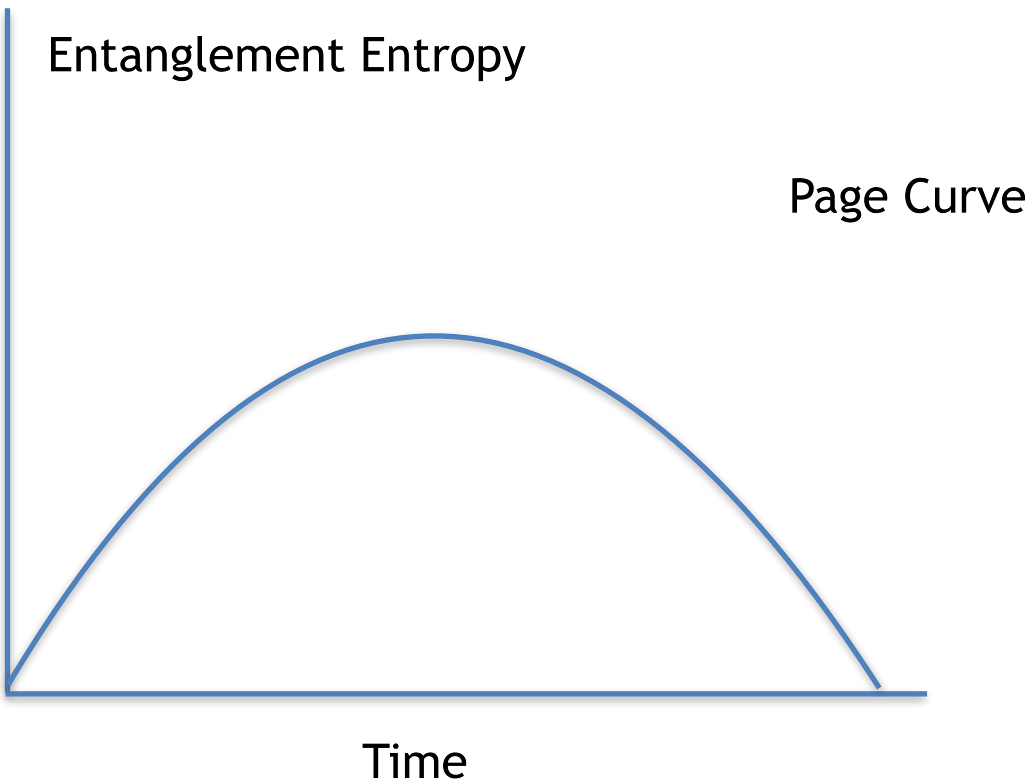
Consider Burning of a Paper:

Once the burning starts, things get evaporated from the paper to the atmosphere, but they are entangled to things remaining inside.

Initially the entanglement entropy which is a measure of entanglement increases.



But, eventually the small correlations in the emissions destroys the entanglement.



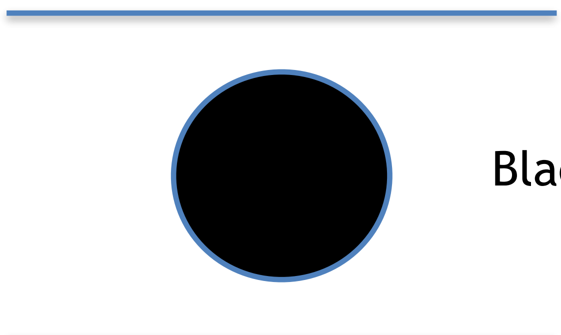
If you start with a initial state which is pure, the final state is also pure.

Quantum Mechanically, buring of the paper is a unitary process….

and this is no longer true for black holes, since there is no correlation between successive emitted Hawking quanta.

The final state is pure thermal radiation….

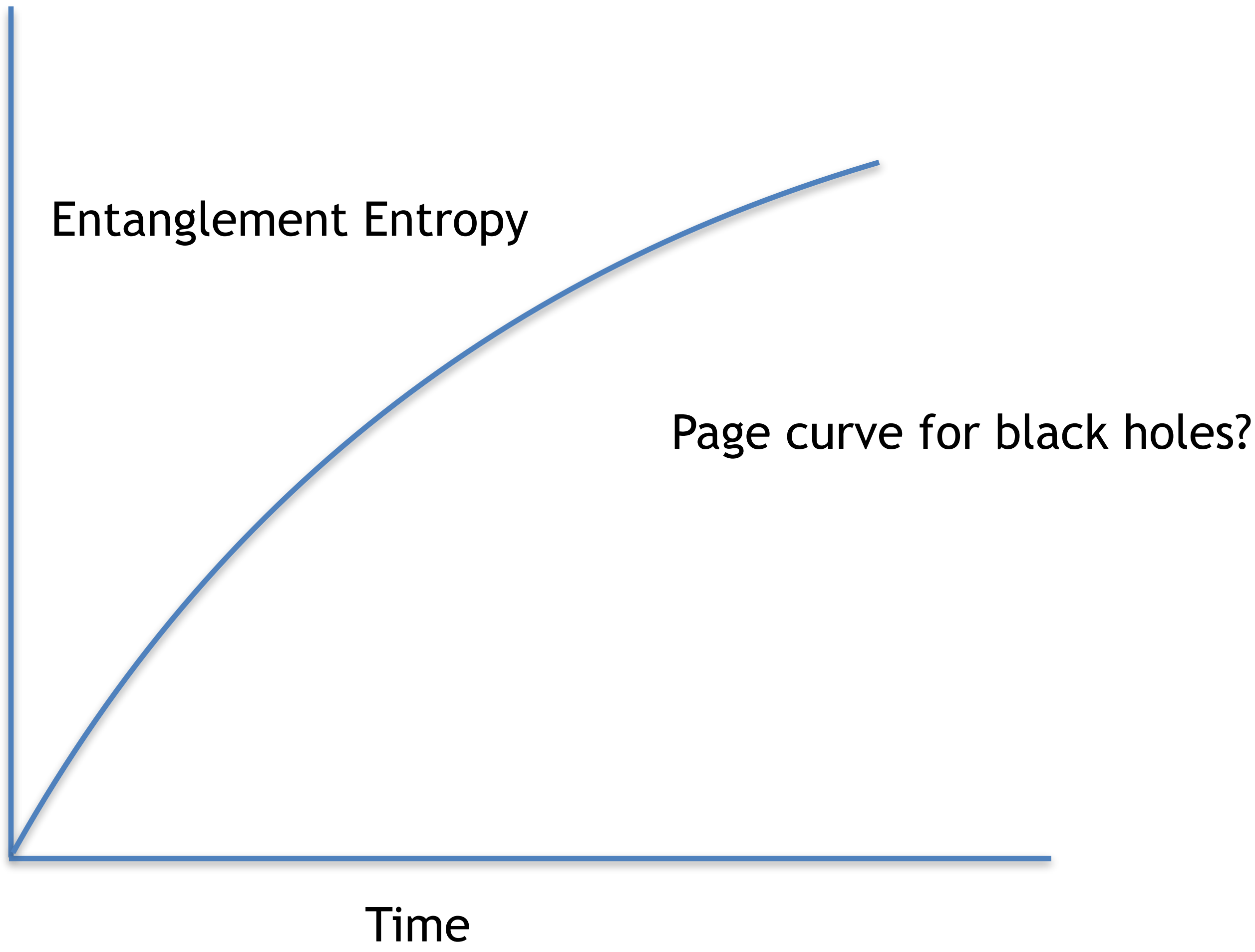
Black hole disappears, pure radiation remains,
the final state in mixed



Black hole forms and evaporates by Hawking Process

Initial Matter in Pure state

Pure to Mixed state: This can NOT be described by an unitary time evolution.
Information loss paradox.


Entanglement Entropy
Page curve for black holes?
Time

Self Radiation correction to Black hole radiation:
(Krauss and Wilczek, Nucl.Phys.B, 1995)

This aims to calculate the correction to black hole ration due to self interaction

A spherically symmetric study shows that in the leading order, the Hawking quanta of energy E moves in a space time of shifted ADM mass M-E.

This introduces correction in the emission probability but can not create any useful correlations.

$$\Gamma(E_1)\,\Gamma(\mathrm{E}_2) = \Gamma(E_1 + E_2)$$

We could calculate the same with a theory of gravity which is an extension of Einstein theory, namely the Gauss Bonnet gravity. Still we get no useful correlation.

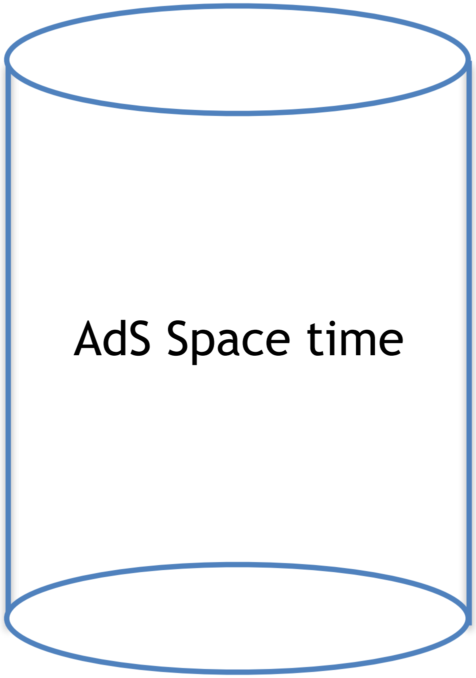
(Fairoos. C, SS, K P Yogendran, to appear soon)

The resolution of information loss paradox is beyond semi classical gravity.

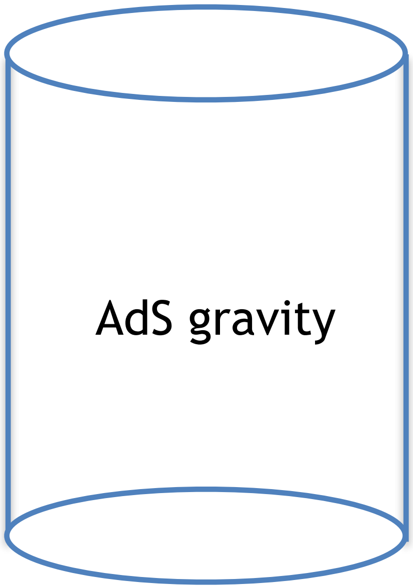
The question of the unitarity of black hole evaporation is a central question which a Quantum Theory of Gravity must resolve.

A possible resolution could be via what is known as the Gauge Gravity duality.

The Anti De sitter space time is a solution of general relativity with a cosmological constant with a time like boundary.



A conformal field theory is a special type of quantum theory with conformal symmetry.



Gravity in AdS = CFT at it's boundary

CFT at the boundary

Consider an evaporating black hole in the bulk, a black hole represents a thermal state in the dual CFT. The dual theory is a well defined unitary QFT.

The evolution of any state is governed by unitary operators. Then there can not be an information loss in the bulk also.

But, this does not explain the end state of the evaporation.

The completely description of black hole evaporation requires quantum theory of gravity.

There could be completely new symmetries which results into new conservation laws leading to unitary descriptions of the black hole evaporation.

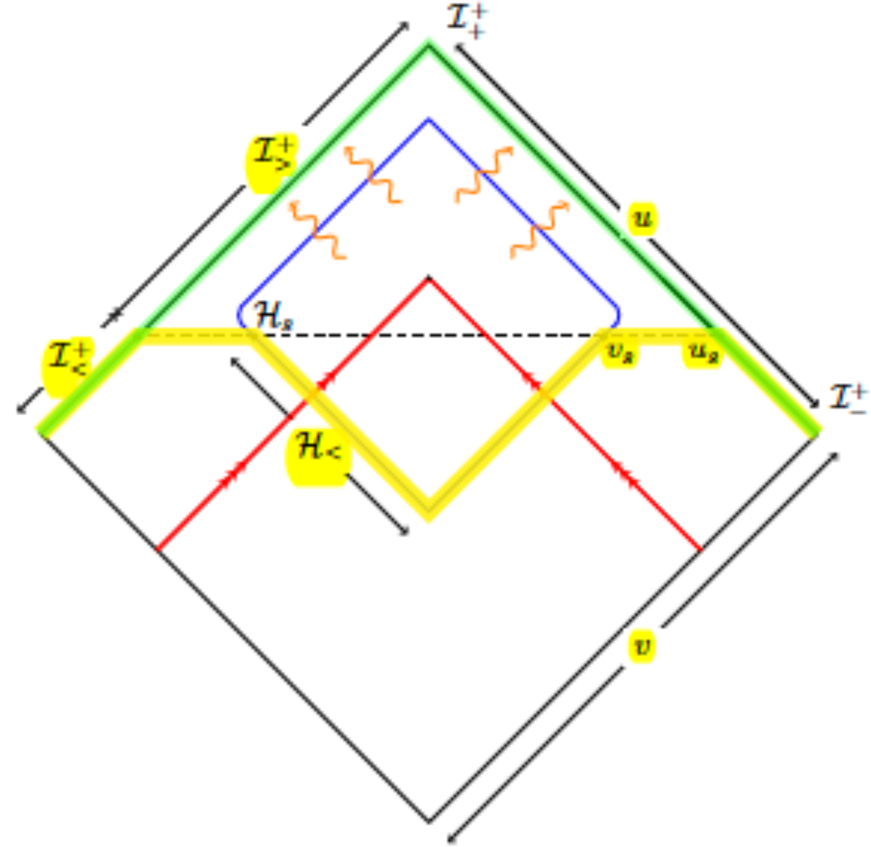
What if there is an infinite dimensional global symmetry….

The global symmetries leads to conserved charges…

An infinite dimensional global symmetry will lead to infinite number of conserved charges.

There is such a symmetry:

The Asymptotic symmetry group in GR: BMS Group



BMS charges acts like quantum hair on black holes and may lead to the restoration of the unitarity.

Hawking, Perry, Strominger, PRL, 2016

Summary:

Classical and Semi classical description of black holes are not complete.

A consistent description of black hole physics requires quantum theory of gravity.

The derivation of semi classical effects like Hawking Radiation and Bekenstein entropy could be a basic check for any theory of quantum gravity.

Black hole Physics = Quantum gravity virtual Laboratory

Thanks